Lecture 3 Summary
Thur 31/07/14

Vocabulary

* normed vector space
* Hölder inequality
* Minkowski inequality

Examples

FUNCTION SPACE : $\mathbb{R}^n = \{x : [1,n]_{\mathbb{Z}} \to \mathbb{R}\} = \{x = (x_1, \ldots, x_n) \mid x_i \in \mathbb{R}\}$

Homework

- What is a seminorm?
- Let $(X, \|\cdot\|)$ be a normed vector space. Define $d : X \times X \to \mathbb{R}_{\geq 0}$ by $d(x,y) = \|y - x\|$. Show that (X, d) is a metric space.
- Let X be a set. Let $\mathcal{Y} = \{f : X \to \mathbb{R}\}$.
  Define $(f+g)(x) = f(x) + g(x)$
  and $(cf)(x) = c \cdot f(x)$
  Show that, with these operations, $\mathcal{Y}$ is a vector space.
- Let $p \in (1, \infty) = \{y \in \mathbb{R} \mid y > 1\}$. Show that $\mathbb{R}^n$ with $\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}}$ is a normed vector space.